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Charge Sign Effect on the Coulomb Logarithm for a Two-Component (e.g., Antihydrogen) Plasma in a Penning Trap

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Abstract. The magnitude of the center-of-mass scattering angle for a collision between two charges of the same sign is the same as the magnitude of the center-of-mass scattering angle for a collision between opposite single charges, everything else being equal. However, this equivalence only applies for a Coulomb interaction of infinite range. If the range of interaction between two charged particles is limited, as in a plasma, the center-of-mass scattering angle acquires a dependence on whether the two particles have the same or opposite charge signs. In the work presented, the effect that the two different charge sign combinations can have on the Coulomb logarithm is assessed by considering a cutoff Coulomb interaction potential. A substantial effect is predicted for neutral or partially neutralized plasmas in Penning traps.

INTRODUCTION

The Coulomb logarithm is a ubiquitous parameter in plasma physics. However, the commonly used expression for the Coulomb logarithm

$$\lambda = \ln\Lambda \qquad (1)$$

is not accurate for small values of Λ where, for a representative binary collision, $\Lambda = 2b_{max}E_c/(kq_1q_2)$ [1]. Here b_{max} is the maximum impact parameter, E_c is the center-of-mass energy, q_1q_2 is the product of the charges, and k is the Coulomb constant [$k = 1/(4\pi\epsilon_0)$ in SI units or $k = 1$ in Gaussian units]. One reason Eq. (1) is not accurate for small values of Λ is that it is an approximation of a more general expression (see below). Another reason is that Eq. (1) considers collisions between two charged point particles using a pure Coulomb interaction potential. The actual range of interaction between two plasma particles is limited. As a result, the Coulomb logarithm should acquire a dependence on the sign of q_1q_2. Here, a new expression for the Coulomb logarithm is derived by considering a cutoff Coulomb interaction potential that may be either attractive or repulsive. The present work extends the results presented in Ref. [1] to include the case of an attractive interaction potential. The new expression may be useful for describing

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kinetic processes in two-component plasmas in nested Penning traps [2,3]. It should be noted that experiments aimed at merging positron and antiproton plasmas in nested Penning traps [4,5] may be able to observe the effect predicted by the theory presented here (or possibly be adversely affected by the effect). Also, the charge sign effect may affect a plasma's stopping power [6]

At this point, it is useful to define various scale lengths and use them to introduce criteria under which the present theory is intended to apply. First, consider two charged particles that experience a pure Coulomb collision. Initially, the charged particles are an infinite distance apart and moving towards each other. The closest possible approach between the particles occurs if they experience a head-on collision. The classical distance of closest approach for a head-on collision is $r_0(0) = kq_1q_2/E_c$, which applies for particles having the same charge sign. If the two particles are oppositely signed and are considered point particles, then $r_0(0) = 0$.

Now consider a one component plasma (OCP) [7] of infinite dimensions. The particles of the OCP are assumed to have a Maxwellian velocity distribution with uniform temperature T and density n. If a collision between two particles in the OCP is approximated as being a pure Coulomb collision, the average minimum distance of closest approach is approximately $r_{00} = kq^2/T$, where q is the charge of each particle and T is temperature in energy units. Another scale length associated with the OCP is the Wigner-Seitz radius, $r_a = [3/(4\pi n)]^{1/3}$, which is approximately the average distance between nearest neighbor particles of the OCP. A third scale length is the Debye length $\lambda_D = [T/(4\pi kq^2 n)]^{1/2} = \sqrt{r_a^3/(3r_{00})}$. The Debye length is the scale length over which the electric field of a charged plasma particle is not canceled (or "shielded") by an opposite electric field produced by the surrounding plasma. (When more than one plasma component is present, all plasma components associated with thermal speeds of the order of or larger than the speed of a particle being shielded contribute to that shielding.) A fourth scale length is the cyclotron radius. Assuming a uniform magnetic field of magnitude B and neglecting the effects of all other particles and fields, the average cyclotron radius of a particle in the OCP is approximately $r_c = \sqrt{mT}/(\ell|q|B)$. Here, m is the mass of a single particle and ℓ is a constant associated with the units used ($\ell = 1$ in SI units and $\ell = 1/c$ in Gaussian units with c as the speed of light).

The coupling parameter for the OCP is defined as $\Gamma = kq^2/(r_a T)$ [7]. The coupling parameter can also be written in terms of the scale lengths defined above: $\Gamma = r_{00}/r_a = (1/3)(r_a/\lambda_D)^2 = (1/3)^{1/3}(r_{00}/\lambda_D)^{2/3}$. The OCP is strongly coupled or strongly correlated if $\Gamma \gg 1$ and weakly coupled or weakly correlated if $\Gamma \ll 1$ [7]. For $\Gamma \geq 2$, the particle positions start to become correlated, and the plasma can exhibit liquid or crystal characteristics [7]. Finally, the OCP is strongly magnetized if $r_c \ll r_{00}$ or weakly magnetized if $r_c \gg r_{00}$ [8]. Below, we find that the transition between strong and weak magnetization occurs at $r_c \approx 0.1 r_{00}$.

In Ref. [8], theoretical expressions for the anisotropic temperature relaxation rate are evaluated for both weakly and strongly magnetized nonneutral electron plasmas. The expressions are the same, except that for a strongly magnetized plasma the usual Coulomb logarithm is replaced by a different term. For weakly magnetized plasmas, the physics

associated with the collision dynamics is contained within the Coulomb logarithm. For strongly magnetized plasmas, the collision dynamics is modified by the magnetic field. In consideration of these things, we hypothesize that rates or time scales that are associated with velocity space scattering processes are describable by expressions that apply to both weakly and strongly magnetized plasmas, except for a term (the Coulomb logarithm) that changes (see also Ref. [2]). For strongly magnetized plasmas, we refer to the term as the Coulomb logarithm although it may not have a logarithmic dependence.

In the work presented here, two interacting plasma components are considered. The cyclotron radius for at least one of the plasma components is assumed to be smaller than the Debye shielding length associated with the presence of both plasma components. Also, each plasma component is weakly correlated and may be either weakly or strongly magnetized.

DERIVATION OF THE COULOMB LOGARITHM

The usual defining expression for the Coulomb logarithm is given by [1]

$$\lambda = \frac{1}{\rho_\perp^2} \int_0^\infty \sin^2\left(\frac{\theta_c}{2}\right) b\, db, \tag{2}$$

where θ_c is the center-of-mass scattering angle for a collision between two plasma particles and $\rho_\perp = kq_1 q_2/(2E_c)$. If we consider the classical center-of-mass scattering angle for a binary collision, then

$$\theta_c = \pi - 2b \int_{r_0}^\infty \frac{dr}{r^2}\left(1 - \frac{V(r)}{E_c} - \frac{b^2}{r^2}\right)^{-1/2}, \tag{3}$$

where $V(r)$ is the interaction potential and

$$0 = 1 - \frac{V(r_0)}{E_c} - \frac{b^2}{r_0^2} \tag{4}$$

identifies the point where $\dot{r}$ equals zero, denoting a turning point (about which the trajectory is symmetric in the center-of-mass frame of reference). Thus, $r_0(b)$ is the classical distance of closest approach for a given impact parameter b. For a pure Coulomb collision, $V(r) = kq_1 q_2/r$ and

$$\sin^2\left(\frac{\theta_c}{2}\right) = \frac{1}{1 + b^2/\rho_\perp^2}. \tag{5}$$

Substituting Eq. (5) into Eq. (2), and introducing a maximum impact parameter b_{max}, yields

$$\lambda = \frac{1}{\rho_\perp^2} \int_0^{b_{max}} \frac{b}{1 + b^2/\rho_\perp^2}\, db = \ln\left(\sqrt{1 + \frac{b_{max}^2}{\rho_\perp^2}}\right) = \ln\left(\sqrt{1 + \Lambda^2}\right). \tag{6}$$

Equation (6) applies regardless of the sign of $\Lambda = 2b_{max}E_c/(kq_1q_2)$ and is symmetric for negative Λ with respect to positive Λ. Note that Eq. (1) is arrived at by taking the limit $\Lambda \gg 1$ and considering only positive values for q_1q_2.

The Coulomb logarithm is now evaluated using a cutoff Coulomb interaction potential, everything else being the same. The cutoff Coulomb interaction potential has the form [1]

$$V(r) = kq_1q_2\left(\frac{1}{r} - \frac{1}{a}\right)\Theta\left(1 - \frac{r}{a}\right), \tag{7}$$

where Θ is the Heavyside step function and a is the cutoff length. If we introduce the variables $y = b/r$, $\Lambda_a = 2aE_c/(kq_1q_2)$, and $\beta = b/a$, the center-of-mass scattering angle is given by

$$\theta_c = \pi - 2\int_0^\beta \frac{dy}{\sqrt{1-y^2}} - 2\int_\beta^{y_0} \frac{dy}{\sqrt{1 + 2/\Lambda_a - 2y/\Lambda_a\beta - y^2}}, \tag{8}$$

where y_0 is found by solving

$$0 = 1 - \frac{2}{\Lambda_a\beta}(y_0 - \beta) - y_0^2. \tag{9}$$

Notice that Λ_a carries the information of the charge signs. Equation (9), for positive Λ_a, yields only one physically admissible solution for y_0 [1]. On the other hand, if Λ_a is allowed to be either negative or positive, multiple possible solutions are obtained. However, the ambiguity vanishes during the calculation of the center-of-mass scattering angle. Upon integration, the Coulomb logarithm for a cutoff Coulomb interaction potential is found to be

$$\lambda(\Lambda_a) = \frac{(1 + \Lambda_a)^2\ln\left[(1 + \Lambda_a)^2\right]}{2(2 + \Lambda_a)^2} - \frac{\Lambda_a}{2(2 + \Lambda_a)}, \tag{10}$$

which applies for both attractive and repulsive interaction potentials.

EFFECT OF THE CHARGE SIGNS

Equation (10) is not symmetric for negative Λ_a with respect to positive Λ_a. This is shown in Fig. 1 where $\Lambda_a = \pm\Lambda$ is used. Unlike the case of the pure Coulomb interaction potential, the Coulomb logarithm for a cutoff Coulomb interaction potential depends on the sign of q_1q_2.

A comparison between the present theory and numerical calculations [8] of the Coulomb logarithm for repulsive interactions within an OCP is presented in Fig. 2, where $\Lambda_a = \pm c_a\Lambda$ is used with a proportionality constant, c_a. (The reader is referred to Ref. [2] for more details on the type of comparison shown in Fig. 2.) Using $\Lambda_a = \pm c_a\Lambda$ amounts to setting $a = c_ab_{max}$, where b_{max} is set equal to the cyclotron radius. $c_a = 2$ is chosen
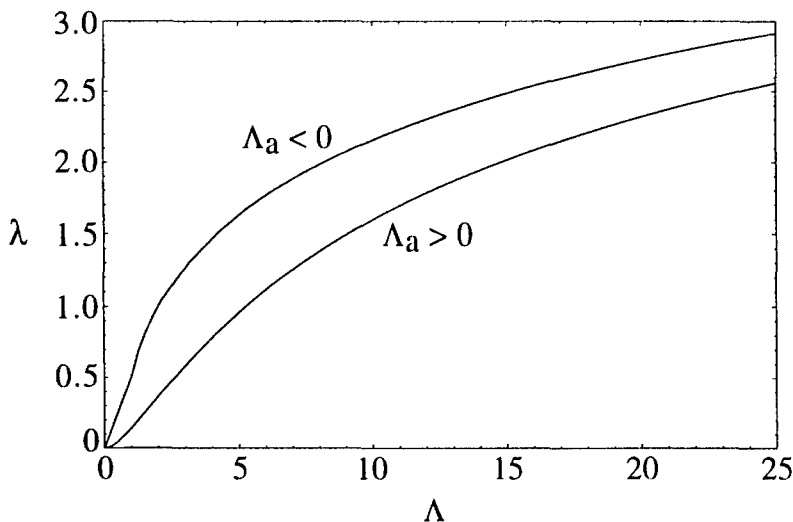
FIGURE 1. Equation (10) using $\Lambda_a = \pm\Lambda$.

because it yields the best fit (by eye) between Eq. (10) with $\Lambda_a > 0$ and the numerical calculations. (Thus, $a \approx 2r_c$.) Equation (10) with $\Lambda_a > 0$ is found to be within 10% of the numerical calculations for $\Lambda > 0.3$ ($\lambda > 0.06$). The transition from disagreement to good agreement between Eq. (10) with $\Lambda_a > 0$ and the numerical calculations may be considered to indicate that the transition from strong to weak magnetization occurs near $\Lambda = 0.3$ within an OCP. For the comparison using $\Lambda_a = -c_a\Lambda$, the prediction for an attractive interaction potential is compared with numerical calculations for a repulsive interaction potential. The result indicates that the charge sign effect is substantial over at least an order of magnitude of Λ values and should be observable in neutral or partially neutralized plasmas in Penning traps.

CONCLUSION

By considering a cutoff Coulomb interaction potential, a new expression for the Coulomb logarithm, Eq. (10), has been derived. The new expression for the Coulomb logarithm exhibits a dependence on whether the interaction potential is repulsive or attractive. For repulsive interactions, Eq. (10) was found to agree with numerical results for an OCP that is weakly magnetized (i.e., an OCP for which $r_c/r_{00} > 0.1$). For attractive interactions, the applicability of Eq. (10) is not expected to be restricted by the ratio r_c/r_{00} associated with each of two oppositely signed plasma components because the classical distance of closest approach can always be less than the smaller of the two cyclotron radii associated with the two components.
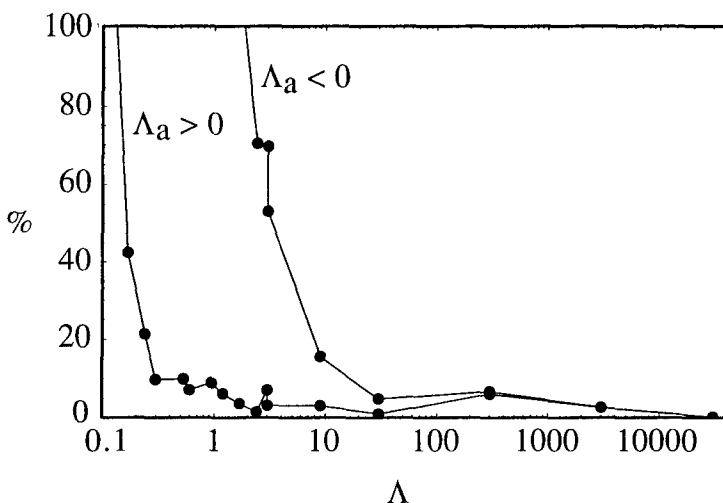
FIGURE 2. The percent difference between Eq. (10) using $\Lambda_a = \pm 2\Lambda$ and numerical calculations. The points are joined by lines to guide the eye.

ACKNOWLEDGMENTS

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